Orthoreconstructions

Nikolai Ivanov Beluhov

1 Introduction

An orthoreconstruction problem asks you to find a cooperative game leading to a position identical with the diagram except for the other player being on the move.

A manoeuvre which accomplishes this task is known as *loss of tempo* in chess composition. The idea of making it the stipulated goal of a problem originated with Luigi Ceriani, an Italian professor of mathematics who published the first orthoreconstruction problems in the late 1940s. Ceriani was followed by Aristide and Marco Ferrari, Karl Fabel, and Luis Alberto Garaza among others.

There is no convention as to who moves first in an orthoreconstruction: unless the stipulation specifies a player, the solver can decide on his own.

Often, the stipulation would specify the minimum number of moves in which the task can be completed. For most orthoreconstruction problems, though, completing the task *at all* is the interesting part in which you work your way through the composer's idea, and boiling the number of moves in the game down to a minimum after that is exhausting and uninteresting. For this reason, I have not specified move counts in my own orthoreconstructions.

Of course, the above does not apply to, say, record problems where the minimum possible length of the solution is an integral part of the composer's design.

2 Classification

As of October 2012, the Chess Problem Database[1] lists 150 problems tagged with the keyword "Ortho-reconstruction". These problems can be divided neatly in two large classes based on the solution's scenario. Each class can then be subdivided more finely according to appropriate criteria.

2.1 Protagonist orthoreconstructions

In a protagonist orthoreconstruction, only one piece – the *protagonist* – is capable of traversing an odd cycle, i.e., of returning to the same square in an odd

[1] At `http://www.softdecc.com/pdb/index.pdb`.

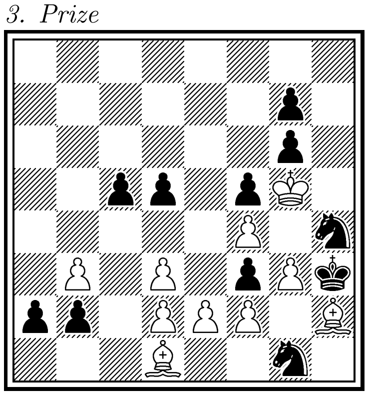
number of moves. The solution follows this piece's adventures as it struggles to actually traverse the above-mentioned odd cycle.

Only one non-pawn – the Knight – is incapable of traversing an odd cycle when left out in the open. In an orthoreconstruction problem, though, neither pawn moves nor captures are possible, and this allows composers to build various structures serving to restrict the pieces' freedom severely.

Orthoreconstructions of the protagonist type can be further classified according to the type of the protagonist and the minimum possible length (in moves) of the odd cycle it aims to traverse.

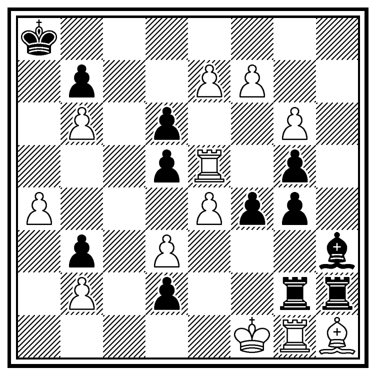
The King enjoys the greatest popularity among all possible protagonists, and three is the most popular cycle length: a King triangulation accounts for a substantial part of the 150 CPDB orthoreconstructions. N1 is an early rendition of this time-tested plot.

1
Hans Theo Kuner
Die Schwalbe 1950
3. Prize



Orthoreconstruction in 68.5 (10+11)

2
Luigi Ceriani
L'Italia Scacchistica 1949



Orthoreconstruction in 18.5 (12+12)

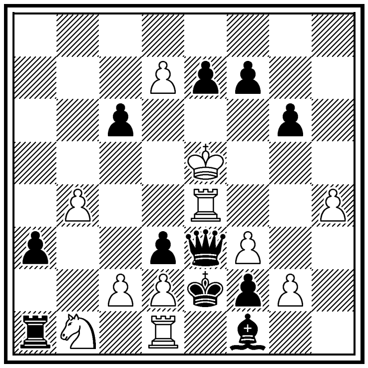
Much longer odd cycles are also possible, as in N2.

All non-pawn pieces except for the Knight can serve as protagonists: N3 features a Rook.

Sometimes, the actual odd-cycling protagonist only plays a very small number of moves throughout the solution whereas other pieces undertake long and perilous journeys in order to enable him to do so. The loyal *sidekick* – or a small number of sidekicks – would be the focus of attention in such a plot, as in N4.

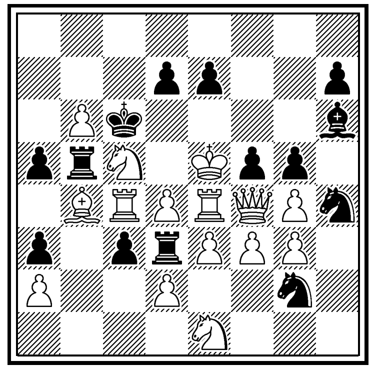
Finally, in a small number of problems such as N5, rather than a single protagonist returning back to its diagram square, there are multiple protagonists of the same type who trade places with one another, an odd number of them

3
Luigi Ceriani
32 Personaggi e 1 Autore 1955



Orthoreconstruction in 26.5 (11+11)

4
Luis Alberto Garaza
problem 1962



Orthoreconstruction in 83.5 (15+14)

traversing odd paths to their new squares.

2.2 Chinese puzzle orthoreconstructions

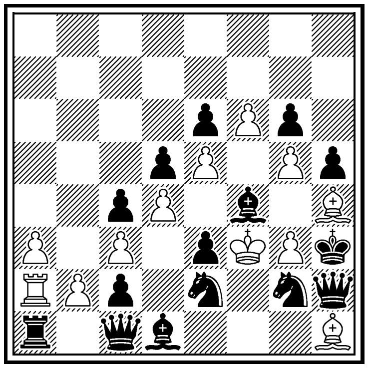
In a Chinese puzzle[2] orthoreconstruction, a number of pieces find themselves enclosed in a cramped box with only a little room left for manoeuvring, the solver dealing with an ensemble cast scenario in which the solution must emerge from the equal-terms interaction of all active pieces rather than be structured around the personal storyline of one central character.

In monochromatic Chinese puzzles such as N6, all units inside the box are of the same colour, with one or more "even" (i.e., incapable of traversing an odd cycle) enemy units ticking away the time outside. In bichromatic Chinese puzzles, the box contains units of both colours. Problems of this second type do sometimes feature even extraneous pieces capable of providing tempos for one or both players should their in-box units be completely blocked, but they do not generally need to, as N7 demonstrates.

In an in-box Chinese puzzle, no unit initially in the box ever leaves it, as in both of N6 and N7. In an extraction Chinese puzzle, at least one piece needs to be extracted from the box at some point in the solution in order to make room

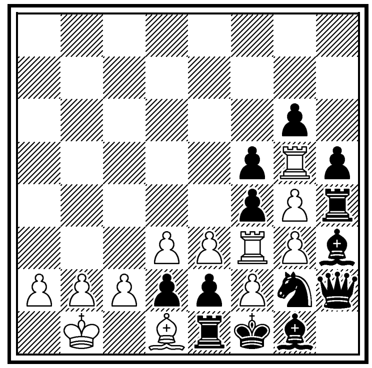
[2]This term for the phenomenon was coined by Guus Rol, who published a number of complex Chinese puzzle retros in the late 1980s: "The designation "chinese puzzle" is [...] my own, coming from the dizzling sensation of endlessly moving units around in a confined space to get to the proper configuration for the next phase. Some chinese puzzles like tangrams give me the same feeling.", `http://www.pairlist.net/pipermail/retros/2009-March/002845.html`.

5
Luigi Ceriani
32 Personaggi e 1 Autore
1955



Orthoreconstruction (12+15)

6
Aristide Ferrari
Marco Ferrari
?



Orthoreconstruction (12+13)

for a parity-switching manoeuvre. N8 is a fine example of a monochromatic extraction Chinese box, and N9 – of a bichromatic one.

3 Mathematics

Unlike most other chess problems, orthoreconstructions are quite susceptible to mathematical treatment.

It is well-known that, during the course of an orthoreconstruction, at least one odd cycle must be traced – either by a single piece, or by a group of identical pieces which trade places in a cyclical fashion as a result. This observation is the solver's guiding light when puzzling over a protagonist orthoreconstruction.

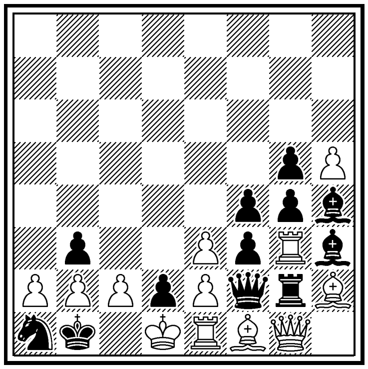
No simple general rule seems to be known in relation with Chinese puzzle orthoreconstructions, and perhaps none exists. The odd cycle principle continues to hold, of course, but in a typical Chinese puzzle it is of no help at all: many pieces will be capable of odd-cycling, and their tracing of the corresponding odd cycles will always be greatly complicated by the need to negotiate the movement of all other active pieces simultaneously.

There is, though, one simple rule of limited application which I have found very useful in dealing with particular Chinese puzzles.

For the following argument, we will represent every position in the game as a 64-item list of the states of all squares on the board, ordered from left to right and from top to bottom, as in "wK-bQ-bK-empty-empty-...-empty". We will consider separate items of the same type, e.g., two "empty"'s, to be different objects.

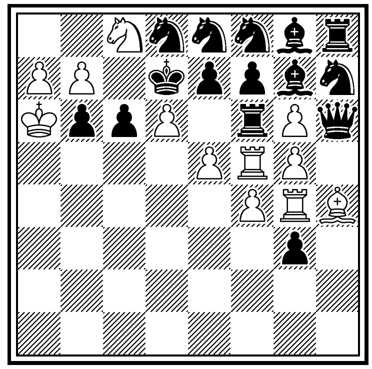
Since no captures or promotions may take place during an orthoreconstruction, every position that we reach from the diagram will be represented by a

7
Marco Ferrari
32 Personaggi e 1 Autore 1955



Orthoreconstruction (12+12)
in 53.5

8
Luigi Ceriani
after A. & M. Ferrari
Fairy Chess Review 1953



Orthoreconstruction (12+15)
in 112.5

permutation of the initial list of square states. Since no castlings are possible either – as they irrevocably change castling rights – every single move will alter this permutation by swapping around two items in the list (one of which will be an “empty”, and the other will not). A transformation of this type is known as a *transposition.*

It is a well-known theorem in combinatorics that only an even number of transpositions can restore a permutation to its original ordering. In an orthoreconstruction, though, we wish to restore the diagram in an odd number of single moves. How is this possible?

The answer is that, in the restored diagram at the end of the solution, a number of list items of the same type will have traded places, producing an identical position but a different permutation of the list. Or, more precisely:

In the restored diagram at the end of the solution, there will always be an even number of list items which have traded places cyclically.

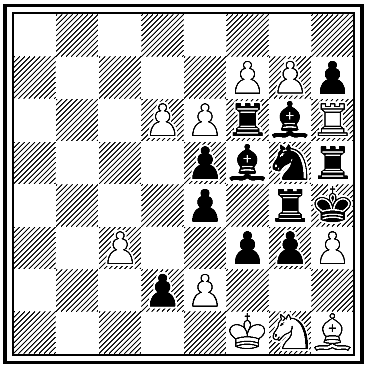
Why not an odd number of items? Because the cyclical exchange of an odd number of objects produces the same result as some even number of transpositions and is therefore of no help in altering parity.

This principle remains unnoticed in protagonist orthoreconstructions because in them many empty items participate in the exchanges and it is precisely those invisible objects that trade places.

In a Chinese puzzle, however, only a very small number of empty items are active, and it is the pieces – most often, a pair of identical ones – that must do the trick.

In N6, there are only two items of the same type capable of an interchange: the two black Rooks. This at once clarifies the composers’ decision to place

9
Luis Alberto Garaza
La Genesi delle Posizioni
1961



Black to move. (11+13)
Orthoreconstruction in 114.5

10
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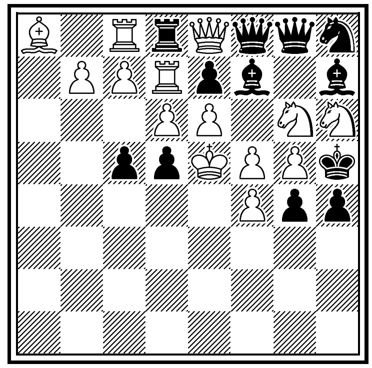
StrateGems 2012



Orthoreconstruction (14+12)

them as far apart as possible, and provides a straightforward path for the solver to follow in an unchartable jungle of possible games. A similar effect, but with white Rooks instead of black, can be observed in N7.

In N8, the principle manifests itself in a different form: a proof of impossibility. Since there are no identical units inside the box which can trade places, all attempts at an in-box solution are doomed to fail. This observation prompts the solver to look for a piece to extract; in the above terminology, an extraction serves to bring a new empty item into play so that an interchange between empty items can occur. Only the bQ is up to the task, and a solver who has focused his efforts on leading her all the way to c7 is firmly on his way to a solution.

In order to demonstrate the impossibility of an in-box orthoreconstruction in N9, a subtler argument is necessary. It is easy to see why the black Rooks are of no use: locked inside the circular alley f4-g4-h4-h5-h6-g6-f6-f5, they can only trade places cyclically, and an exchange involving an odd number of participants – three, in this case – would not alter parity. But why cannot the two black Bishops switch places?

Label the bBf5 X and the bBg6 Y. X moves first to g4, and after that the two Bishops and the paralyzed bNg5 in the center of the box form a wall which the empty space cannot jump over: for the next bB move, either X must backtrack or Y must move to f5. The same effect continues to occur: occasional backtracking omitted, the sequence of the two black Bishops' moves can only be Xf5-g4, Yg6-f5, Xg4-h5, Yf5-g4, Xh5-g6, Yg4-h5, Xg6-f5, Yh5-g6, Xf5-g4, etc., the same eight steps repeated indefinitely. Each time the black Bishops return to their diagram squares, X returns to f5 and Y returns to g6, an interchange being infeasible.

4 Problems

During the summer of 2012, I composed a number of problems whose scenarios naturally grew out of the above ideas and observations.

My goal was to point the solver's attention to the mathematical aspects of the genre: the existence of general principles and the possibility of rigorous proofs of insolubility. These concepts become particularly tangible when one is faced with the task of discovering the unique position – out of a number of near-identical ones – which would allow an orthoreconstruction.

In N10, this is achieved by means internal to the genre. The game opens with 1.Rc8-b8 Rd8-c8 2.Qe8-d8..., and after that one has to choose between 2. ... Qf8-e8 and 2. ... Bf7-e8. In both cases, the solution must proceed with an orthoreconstruction of the newly animated Chinese puzzle in the north-east corner, after which the first four single moves are replayed in reverse. Only one of the two possible Chinese puzzles allows an orthoreconstruction, though!

It seems highly unlikely that a scheme of this type could lead to much more than a three- or four-way choice. Fortunately, position reconstruction ideas from retrograde analysis come in delightfully handy at this point.

In N11, the contents of the box in a Chinese puzzle problem are presented as a rebus. There are 36 ways to assign pieces to letters and obtain a legal position, 24 of them resulting in functional Chinese puzzles; and yet, only one makes an orthoreconstruction possible.

11
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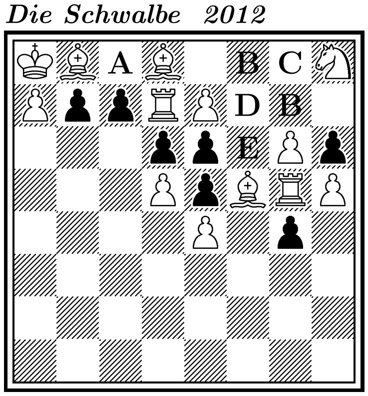
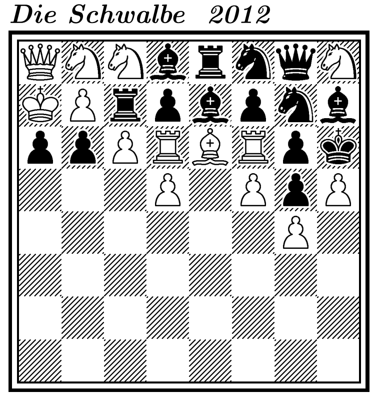
Die Schwalbe 2012



Letters stand for non-pawn black pieces. Orthoreconstruction (13+7)

12
Nikolai Beluhov
Die Schwalbe 2012

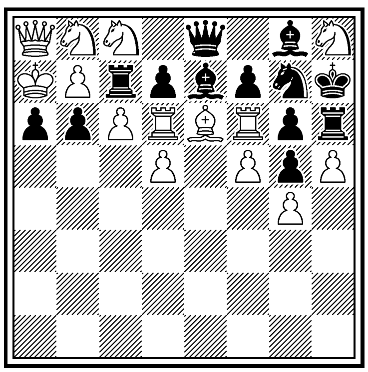


Remove two non-pawn black pieces for an orthoreconstruction with Black on the move (14+15)

In N12, there are 28 ways to remove a pair of non-pawn black pieces and leave behind a legal position. Eighteen of these pairs produce functional Chinese puzzles; and, once again, only one allows for a subsequent orthoreconstruction.

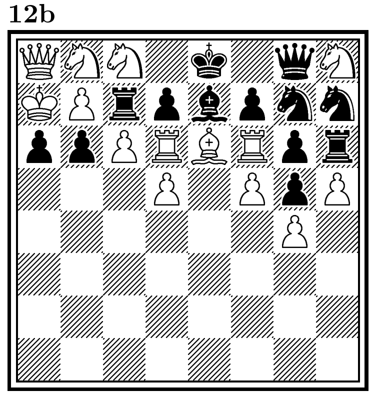
While working on N12, I discovered two Chinese puzzles set on the same stage which possess particularly long and complicated resolutions, N12a and N12b. Together with N12 with the two superfluous units removed, they constitute the *only* non-trivial Chinese puzzle orthoreconstructions that I know of with more than one empty in-box square. (The deciphered N11 may appear at first to be a double-vacancy bichromatic Chinese puzzle, but it is actually a single-vacancy monochromatic one: since no interaction occurs between the wNh8 and the rest of the box, the wN is better construed as an extra-box unit penduluming on the two extra-box squares f7 and h8. This is not the case with N12 and N12a-b.)

12a



Orthoreconstruction (14+13)

12b



Orthoreconstruction (14+13)

In N13, the solver must decide both on the set of pieces to populate the box and on their positioning inside it.

N13 was the final problem in a series of unique-solution addition orthoreconstructions which started out as a purely mathematical investigation: Imagine a black Queen immersed in an endless fluid composed of identical white pieces, one square left empty for manoeuvring. Is an orthoreconstruction possible in such a setting?

With a fluid composed solely of Rooks, a 2×2 playing area suffices; N14 is one rendition of this fact.

With a fluid composed of Knights, a playing area of size at least 5×5 is necessary. Quite surprisingly, the starting position on the smallest possible board is unique modulo rotation and reflection, and this makes for a very nice puzzle, N15.

With a fluid composed of Bishops, no orthoreconstruction is possible regardless of the starting position: since the black Queen must step to an orthogonally adjacent square on her every move, both she and the white Bishops must play an even number of single moves in order to re-occupy their original squares.

13
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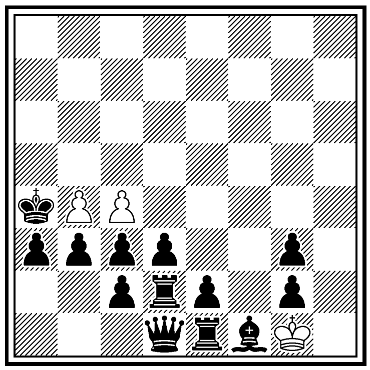
Original



Place four pieces (3+13)
inside the south-west box for
an orthoreconstruction

14
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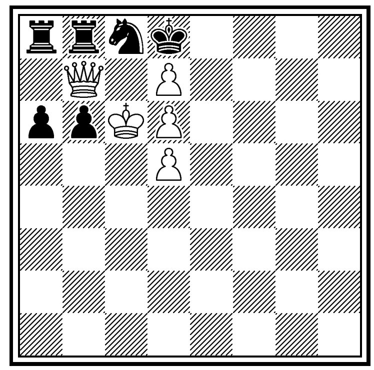
Original



White to move. (5+6)
Orthoreconstruction

We can introduce some impurity, though, by replacing a small number of Bishops with white pieces of other types. One Knight is insufficient; two Knights lead to N16, a fairy orthoreconstruction with a surprisingly long resolution; and one Rook leads to another addition puzzle whose solution is unique modulo rotation and reflection, N17.

The discovery that there exist Chinese puzzles whose contents can be placed inside the box in a unique way so as to allow an orthoreconstruction was quite unexpected for me. N13 was my attempt at an orthodox presentation of the idea; I was not able to achieve a larger box, and would be delighted to see one.

The "follow-the-leader" fairy condition in N18 has the potential of clearing away a lot of technical difficulties in the construction of more complex single-vacancy bichromatic Chinese puzzles. It also has one curious side-effect: the first move determines the shape of the box inside which all subsequent action is to take place.

I did not attempt orthoreconstructions involving uncoloured pieces or dissected boards to be put back together, and certainly many other ideas did not occur to me at all: there is still a lot to explore.

Apart from the above experimental work, I also did a small number of orthodox orthoreconstructions. N19 shares a lot of ideas with the deciphered N11, yet requires much more complicated manoeuvring; unfortunately, I was not able to find a legal implementation of the scheme. N20 is remarkable for its unexpectedly long solution given the innocent-looking diagram. N21 is very easy to solve, yet presents a rather appealing pattern. Finally, N22 is a novel twist on the extraction scenario.

15

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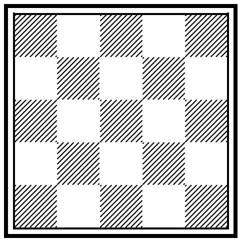
Original



Place one black Queen and 23 white Knights on a 5×5 chessboard for an orthoreconstruction

16

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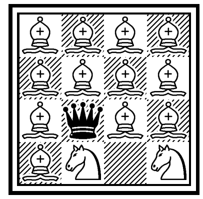
Original



Orthoreconstruction (14+1)

17

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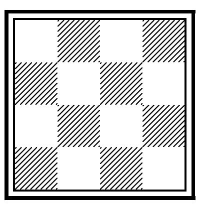
Original



Place one black Queen, one white Rook and 13 white Bishops on a 4×4 chessboard for an orthoreconstruction

18

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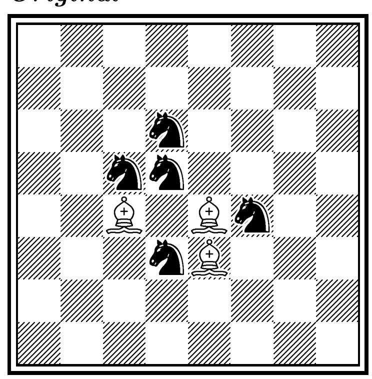
Original



Each square vacated by a piece must be taken up by another piece on the following move. Orthoreconstruction (3+5)

19
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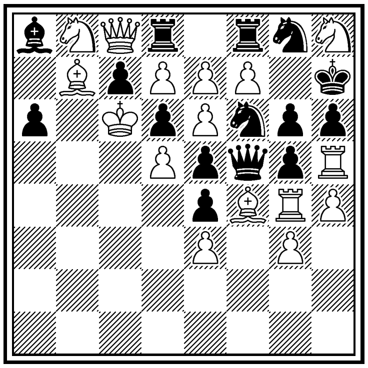
Original



Orthoreconstruction (16+15)
(Illegal position)

20
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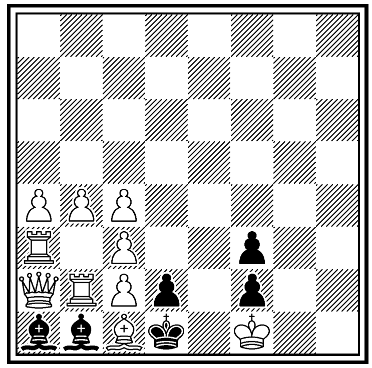
Original



Orthoreconstruction (10+6)

21
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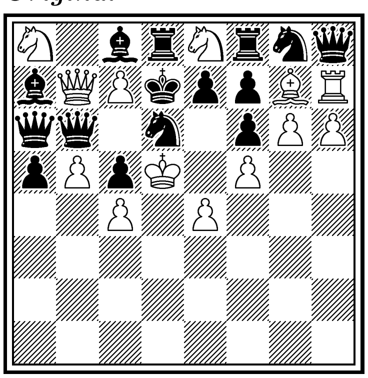
Original



White to move. (13+15)
Orthoreconstruction

22
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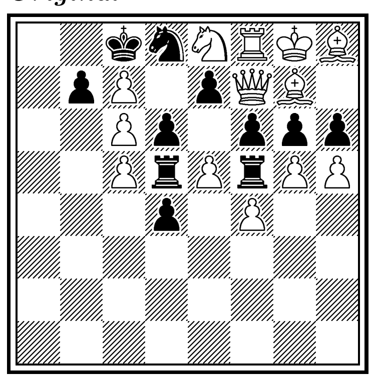
Original



Black to move. (13+11)
Orthoreconstruction

5 Acknowledgements

I am deeply grateful to Mario Richter who provided many valuable comments on my problems and whose computer program `rawbats` was of indispensable help in my work on both orthoreconstructions in general and the present text in particular. This program was used to test almost all of the problems included here, as well as to generate all the detailed solutions and solution statistics. Furthermore, the central mechanism in N12 was inspired by a cook which `rawbats` found in an earlier version of N11, and N22 evolved out of a flaw the program revealed in an earlier version of N21.

I am also grateful to Andrey Frolkin who read the first draft of the text and provided a lot of helpful advice.

6 Solutions

N1: The bK (the protagonist) travels to a1; the wK travels to c3 in order to let the bK pass him by; the bK travels to h7, triangulates by means of h7-g8-h8-h7, and all is repeated in reverse.

N2: With Black on the move, the bK (the protagonist) travels to d7, traces the 13-move cycle d7-e6-f6-g7-h6-h5-h4-g3-f3-e3-d4-c5-c6-d7, and returns back to a8.

N3: The bRa1 (the protagonist) travels to h3, loses tempo by means of h3-h2-h1-h3, and returns back to a1. The shortest solution is unique modulo the direction of time. (I.e., there are only two shortest solutions and each one of them coincides with the other one played backwards.)

N4: The wN (the sidekick) travels to b2; a black Knight travels to h1, allowing the wN to pass it by, then retreats to a4; the wN returns to b2; the second bN travels to h1, allowing the wN to pass it by; the wN, not hindered by the bNs any more, travels to f6 in order to screen for the bB (the protagonist); the bB loses tempo by means of h6-f8-g7-h6, and all is repeated in reverse.

N5: The bQh2 travels to f5; the bKh3 travels to d3; the bQc1 travels to g1; the bKd3 travels to e1; the bQf5 takes her twin's place on c1; the bQg1 travels to f5; the bKe1 returns to h3; and the bQf5 takes her twin's place on h2, having traversed an odd path. The shortest solution takes up 32.5 moves.

N6: One of the 16 shortest solutions, White only playing his King: 1.Qh2-h1 2.Bg1-h2 3.Kf1-g1 4.Re1-f1 5.Ng2-e1 6.Qh1-g2 7.Kg1-h1 8.Rf1-g1 9.Qg2-f1 10.Rg1-g2 11.Bh2-g1 12.Rg2-h2 13.Bh3-g2 14.Rh2-h3 15.Bg1-h2 16.Kh1-g1 17.Bg2-h1 18.Ne1-g2 19.Qf1-e1 20.Kg1-f1 21.Bh2-g1 22.Rh3-h2 23.Rh4-h3 24.Ng2-h4 25.Bh1-g2 26.Rh2-h1 27.Rh3-h2 28.Bg2-h3 29.Rh2-g2 30.Bg1-h2 31.Rg2-g1 32.Kf1-g2 33.Rg1-f1 34.Bh2-g1 35.Rh1-h2 36.Kg2-h1 37.Bh3-g2 38.Rh2-h3 39.Kh1-h2 40.Bg2-h1 41.Nh4-g2 42.Rh3-h4 43.Kh2-h3 44.Bg1-h2 45.Rf1-g1 46.Qe1-f1 47.Ng2-e1 48.Qf1-g2 49.Rg1-f1 50.Qg2-g1 51.Bh1-g2 52.Qg1-h1 53.Bh2-g1 54.Kh3-h2 55.Bg2-h3 56.Kh2-g2 57.Bg1-h2 58.Kg2-g1 59.Ne1-g2 60.Rf1-e1 61.Kg1-f1 62.Bh2-g1 63.Qh1-h2.

N7: The shortest solution is unique modulo the direction of time: 1.Qg1-h1 Rg2-g1 2.Rg3-g2 Qf2-g3 3.Rg2-f2 Qg3-g2 4.Bh2-g3 Qg2-h2 5.Qh1-g2 Rg1-h1 6.Qg2-g1 Qh2-g2 7.Qg1-h2 Qg2-g1 8.Rf2-g2 Qg1-f2 9.Rg2-g1 Bh3-g2 10.Qh2-h3 Rh1-h2 11.Rg1-h1 Qf2-g1 12.Bg3-f2 Bh4-g3 13.Qh3-h4 Rh2-h3 14.Rh1-h2 Bg2-h1 15.Bf1-g2 Qg1-f1 16.Bf2-g1 Bg3-f2 17.Qh4-g3 Rh3-h4 18.Bg2-h3 Qf1-g2 19.Re1-f1 Bf2-e1 20.Bg1-f2 Qg2-g1 21.Rh2-g2 Qg1-h2 22.Rg2-g1 Bh1-g2 23.Rg1-h1 Qh2-g1 24.Rh1-h2 Qg1-h1 25.Rf1-g1 Bg2-f1 26.Rh2-g2 Qh1-h2 27.Rg1-h1 Qh2-g1 28.Rh1-h2 Qg1-h1 29.Rg2-g1 Bf1-g2 30.Rg1-f1 Qh1-g1 31.Rh2-h1 Qg1-h2 32.Rh1-g1 Bg2-h1 33.Rg1-g2 Qh2-g1 34.Rg2-h2 Qg1-g2 35.Bf2-g1 Be1-f2 36.Rf1-e1 Qg2-f1 37.Bh3-g2 Rh4-h3 38.Qg3-h4 Bf2-g3 39.Bg1-f2 Qf1-g1 40.Bg2-f1 Bh1-g2 41.Rh2-h1 Rh3-h2 42.Qh4-h3 Bg3-h4 43.Bf2-g3 Qg1-f2 44.Rh1-g1 Rh2-h1 45.Qh3-h2 Bg2-h3 46.Rg1-g2 Qf2-g1 47.Rg2-f2 Qg1-g2 48.Qh2-g1 Qg2-h2 49.Qg1-g2 Rh1-g1 50.Qg2-h1 Qh2-g2 51.Bg3-h2 Qg2-g3 52.Rf2-g2 Qg3-f2 53.Rg2-g3 Rg1-g2 54.Qh1-g1.

N8: One of the 36 shortest solutions, White only playing his Rf5: 1. Rf6-e6 2.Qh6-h5 3.Bg7-h6 4.Ne8-g7 5.Kd7-e8 6.Nf8-d7 7.Ke8-f8 8.Ng7-e8 9.Kf8-g7 10.Nh7-f8 11.Rh8-h7 12.Kg7-h8 13.Bh6-g7 14.Rh7-h6 15.Bg8-h7 16.Kh8-g8 17.Bg7-h8 18.Kg8-g7 19.Bh7-g8 20.Nf8-h7 21.Kg7-f8 22.Ne8-g7 23.Kf8-e8 24.Nh7-f8 25.Rh6-h7 26.Qh5-h6 27.Ng7-h5 28.Qh6-g7 29.Rh7-h6 30.Qg7-h7 31.Nh5-g7 32.Rh6-h5 33.Qh7-h6 34.Nf8-h7 35.Ke8-f8 36.Ng7-e8 37.Kf8-g7 38.Nh7-f8 39.Bg8-h7 40.Kg7-g8 41.Bh8-g7 42.Kg8-h8 43.Bh7-g8 44.Qh6-h7 45.Bg7-h6 46.Qh7-g7 47.Nf8-h7 48.Qg7-f8 49.Ne8-g7 50.Qf8-e8 51.Nd7-f8 52.Qe8-d7 53.Qd7-c7... The bQ has been successfully extracted. 54.Ng7-e8 55.Bh6-g7 56.Nf8-d7 57.Bg7-f8 58.Bf8-h6 59.Ne8-g7 60.Nd7-f8 61.Qc7-d7... The bQ has returned, and moves 1–52 are now replayed in reverse.

N9: One of the four shortest solutions: 1. ... Rg4-f4 2.Bh1-g2 Bf5-g4 3.Bg2-h1 Bg6-f5 4.Rh6-g6 Rh5-h6 5.Bh1-g2 Bg4-h5 6.Bg2-h1 Bf5-g4 7.Bh1-g2 Rf6-f5 8.Rg6-f6 Bh5-g6 9.Bg2-h1 Bg4-h5 10.Bh1-g2 Rf4-g4 11.Bg2-h1 Rf5-f4 12.Bh1-g2 Bg6-f5 13.Bg2-h1 Bh5-g6 14.Bh1-g2 Kh4-h5 15.Bg2-h1 Rg4-h4 16.Bh1-g2 Bf5-g4 17.Bg2-h1 Bg6-f5 18.Bh1-g2 Rh6-g6 19.Bg2-h1 Kh5-h6 20.Bh1-g2 Bg4-h5 21.Bg2-h1 Bf5-g4 22.Rf6-f5 Rg6-f6 23.Bh1-g2 Bh5-g6 24.Bg2-h1 Bg4-h5 25.Bh1-g2 Rf4-g4 26.Rf5-f4 Bg6-f5 27.Bg2-h1 Bh5-g6 28.Bh1-g2 Rh4-h5 29.Bg2-h1 Rg4-h4 30.Bh1-g2 Bf5-g4 31.Bg2-h1 Bg6-f5 32.Bh1-g2 Kh6-g6 33.Bg2-h1 Rh5-h6 34.Bh1-g2 Bg4-h5 35.Bg2-h1 Bf5-g4 36.Bh1-g2 Rf6-f5 37.Bg2-h1 Kg6-f6 38.Bh1-g2 Bh5-g6 39.Bg2-h1 Bg4-h5 40.Rf4-g4 Rf5-f4 41.Bh1-g2 Bg6-f5 42.Bg2-h1 Bh5-g6 43.Bh1-g2 Rh4-h5 44.Rg4-h4 Bf5-g4 45.Bg2-h1 Bg6-f5 46.Bh1-g2 Rh6-g6 47.Bg2-h1 Rh5-h6 48.Bh1-g2 Bg4-h5 49.Bg2-h1 Bf5-g4 50.Bh1-g2 Kf6-f5 51.Bg2-h1 Rg6-f6 52.Bh1-g2 Bh5-g6 53.Bg2-h1 Bg4-h5 54.Bh1-g2 Rf4-g4 55.Bg2-h1 Kf5-f4 56.Bh1-g2 Kf4-e3... The bK has been successfully extracted. 57.Bg2-h1 Rf6-f5 58.Bh1-g2 Rf5-f4 59.Bg2-h1 Rf4-f6 60.Bh1-g2 Ke3-f4... The bK has returned, and moves 1–55 are now replayed in reverse.

N10: Suppose that we played 2. ... Qf8-e8. Now the two black Bishops must switch places, so at some point during the solution they will occupy either f7 and g8, or g6 and h7. Suppose, without loss of generality, that they come to occupy f7 and g8. At the moment right after the bBs take their places, one wN is blocked on h6 and the other one must step from f8 to either g6 or h7 (since one of these two squares has just been vacated). Alas, no black piece could then take up the wN's place on f8: the bBs and the wNh6 prevent the bQ from reaching f8, and Black has no available next move!

The shortest solution is unique modulo the direction of time: 2. ... Bf7-e8 3.Nh6-f7 Qf8-h6 4.Ng6-f8 Qh6-g6 5.Nf7-h6 Qg6-f7 6.Nf8-g6 Qg8-f8 7.Nh6-g8 Qf8-h6 8.Ng6-f8 Qh6-g6 9.Ng8-h6 Qf7-g8 10.Nh6-f7 Qg6-h6 11.Nf8-g6 Qh6-f8 12.Nf7-h6 Be8-f7 13.Qd8-e8 Rc8-d8 14.Rb8-c8.

N11: During the course of the solution, the two **B**'s must switch places.

Suppose, then, that **B** is a black Rook. The switch will have to proceed as follows, modulo the direction of time: (1) a black Rook goes to e8; (2) the other black Rook goes to f8, then to f6 (via f7 while the white Knight is on h8); (3) the bRe8 steps back on f8.

Right after step (3), only a black Knight coming from g7 could move to the square vacated by the black Rook. But this Knight would then have been blocked on g7 all the while since step (1), and no black piece could have vacated the f6-square for the black Rook to move to during step (2): a contradiction.

Therefore, **B** is a black Knight. (Promotees are ruled out by legality considerations.) In this case, the exchange would have to proceed as follows, once again modulo the direction of time: (1) a black Knight goes to g8; (2) another black Knight goes from h7 to f6; (3) and then from f6 to e8.

A black light-squared Bishop inside the box would interfere with step (2), and a black King inside the box would be blocked after step (2) and would interfere with the transfer of the empty item from h7 to e8 prior to step (3).

That said, **A** is the King, **C** is a Rook, **D** is the Queen, and **E** is the dark-squared Bishop. The shortest solution is unique modulo the direction of time, the white Knight oscillating on f7 and h8: 1.Qf7-e8 2.Nf8-h7 3.Qe8-f8 4.Ng7-e8 5.Qf8-g7 6.Rg8-f8 7.Qg7-g8 8.Bf6-g7 9.Ne8-f6 10.Rf8-e8 11.Bg7-f8 12.Qg8-g7 13.Nf6-g8 14.Nh7-f6 15.Qg7-h7 16.Bf8-g7 17.Re8-f8 18.Nf6-e8 19.Ng8-f6 20.Qh7-g8 21.Nf6-h7 22.Bg7-f6 23.Qg8-g7 24.Rf8-g8 25.Qg7-f8 26.Ne8-g7 27.Qf8-e8 28.Nh7-f8 29.Qe8-f7.

N12: After the two superfluous pieces are removed, two in-box items will have to be interchanged. It is not difficult to see that only the two empty items could do; label the one switched with the white Knight on its first move X, and the other one Y.

While oscillating on a fixed pair of squares, the white Knight engages one of the empty items completely, so that the black pieces only have the other one at their disposal. Whenever the white Knight changes its oscillation space (the three possibilities being c8–e7, e7–g8, and g8–h6) it must also change the empty item it engages. (Since no black piece could help transfer an empty item between c8 and g8, or between e7 and h6.) It follows that whenever the white Knight oscillates on e7 and g8 the empty item engaged is Y, and in all other cases – X.

The white Knight's last move in the solution, then, leaves X on e7. If Black's last move was the inverse of his first move or unrelated to e7, X would be left in its original place and no interchange would have taken place. In order for Black to be capable of differing first and last moves from and to e7, we need to leave a black piece (the dark-squared Bishop) there, and to remove the bBd8 and the bNf8. This accounts for the first part of the stipulation.

The actual orthoreconstruction must proceed according the following plan (or its time-reverse): the dark-squared Bishop steps from e7 to d8; the white Knight's oscillation space is gradually shifted from c8-e7 to g8-h6 in order to allow the dark-squared Bishop to visit e7; the dark-squared Bishop is transferred to the right-hand part of the cage via e7; the white Knight's oscillation space is shifted back to c8-e7; and the dark-squared Bishop steps from f8 back to e7.

One of the four shortest solutions, all of them implementing the above plan rather smoothly: 1. ... Be7-d8 2.Nc8-e7 Re8-f8 3.Ne7-c8 Ng7-e8 4.Nc8-e7 Qg8-g7 5.Ne7-c8 Bh7-g8 6.Nc8-e7 Qg7-h7 7.Ne7-c8 Kh6-g7 8.Nc8-e7 Qh7-h6 9.Ne7-c8 Bg8-h7 10.Nc8-e7 Rc7-c8 11.Ne7-g8 Ne8-c7 12.Ng8-e7 Rf8-e8 13.Ne7-g8 Kg7-f8 14.Ng8-e7 Qh6-g7 15.Ne7-g8 Bd8-e7 16.Ng8-h6 Re8-d8 17.Nh6-g8 Kf8-e8 18.Ng8-h6 Be7-f8 19.Nh6-g8 Qg7-h6 20.Ng8-e7 Bf8-g7 21.Ne7-g8 Ke8-f8 22.Ng8-e7 Nc7-e8 23.Ne7-g8 Rc8-c7 24.Ng8-e7 Bh7-g8 25.Ne7-c8 Qh6-h7 26.Nc8-e7 Bg7-h6 27.Ne7-c8 Qh7-g7 28.Nc8-e7 Bg8-h7 29.Ne7-c8 Qg7-g8 30.Nc8-e7 Kf8-g7 31.Ne7-c8 Qg8-f8 32.Nc8-e7 Bh7-g8 33.Ne7-c8 Kg7-h7 34.Nc8-e7 Bh6-g7 35.Ne7-c8 Kh7-h6 36.Nc8-e7 Bg8-h7 37.Ne7-c8 Qf8-g8 38.Nc8-e7 Bg7-f8 39.Ne7-c8 Ne8-g7 40.Nc8-e7 Rd8-e8 41.Ne7-c8 Bf8-e7.

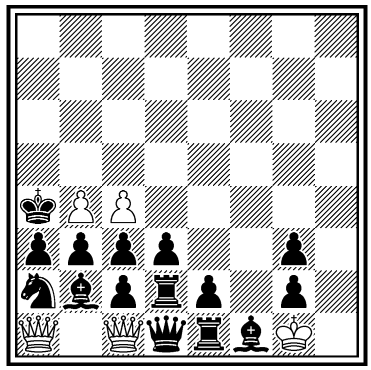
N12a: This time, the wN's oscillation space needs to be shifted to and fro a few more times. One of the 12 shortest solutions: 1. ... Be7-d8 2.Nc8-e7 Qe8-f8 3.Ne7-c8 Ng7-e8 4.Nc8-e7 Kh7-g7 5.Ne7-c8 Bg8-h7 6.Nc8-e7 Qf8-g8 7.Ne7-c8 Kg7-f8 8.Nc8-e7 Qg8-g7 9.Ne7-c8 Bh7-g8 10.Nc8-e7 Rh6-h7 11.Ne7-c8 Qg7-h6 12.Nc8-e7 Rh7-g7 13.Ne7-c8 Bg8-h7 14.Nc8-e7 Rg7-g8 15.Ne7-c8 Kf8-g7 16.Nc8-e7 Rg8-f8 17.Ne7-g8 Rc7-c8 18.Ng8-e7 Ne8-c7 19.Ne7-g8 Rf8-e8 20.Ng8-e7 Kg7-f8 21.Ne7-g8 Qh6-g7 22.Ng8-h6 Bd8-e7 23.Nh6-g8 Re8-d8 24.Ng8-h6 Kf8-e8 25.Nh6-g8 Be7-f8 26.Ng8-e7 Qg7-h6 27.Ne7-g8 Bf8-g7 28.Ng8-e7 Ke8-f8 29.Ne7-g8 Nc7-e8 30.Ng8-e7 Rc8-c7 31.Ne7-c8 Bh7-g8 32.Nc8-e7 Qh6-h7 33.Ne7-c8 Bg7-h6 34.Nc8-e7 Qh7-g7 35.Ne7-c8 Bg8-h7 36.Nc8-e7 Rc7-c8 37.Ne7-g8 Ne8-c7 38.Ng8-e7 Kf8-e8 39.Ne7-g8 Qg7-f8 40.Ng8-e7 Bh6-g7 41.Ne7-g8 Qf8-e7 42.Ng8-h6 Ke8-f8 43.Nh6-g8 Rd8-e8 44.Ng8-h6 Qe7-d8 45.Nh6-g8 Bg7-h6 46.Ng8-e7 Kf8-g7 47.Ne7-g8 Re8-f8 48.Ng8-e7 Nc7-e8 49.Ne7-g8 Rc8-c7 50.Ng8-e7 Bh7-g8 51.Ne7-c8 Kg7-h7 52.Nc8-e7 Bh6-g7 53.Ne7-c8 Kh7-h6 54.Nc8-e7 Bg8-h7 55.Ne7-c8 Rf8-g8 56.Nc8-e7 Bg7-f8 57.Ne7-c8 Rg8-g7 58.Nc8-e7 Bh7-g8 59.Ne7-c8 Rg7-h7 60.Nc8-e7 Kh6-g7 61.Ne7-c8 Rh7-h6 62.Nc8-e7 Kg7-h7 63.Ne7-c8 Ne8-g7 64.Nc8-e7 Qd8-e8 65.Ne7-c8 Bf8-e7.

N12b: Here, the wN's oscillation space shifting pattern is even more erratic. One of the four shortest solutions: 1. ... Be7-d8 2.Nc8-e7 Ke8-f8 3.Ne7-c8 Ng7-e8 4.Nc8-e7 Kf8-g7 5.Ne7-c8 Nh7-f8 6.Nc8-e7 Rh6-h7 7.Ne7-c8 Kg7-h6 8.Nc8-e7 Rh7-g7 9.Ne7-c8 Qg8-h7 10.Nc8-e7 Rg7-g8 11.Ne7-c8 Kh6-g7 12.Nc8-e7 Qh7-h6 13.Ne7-c8 Nf8-h7 14.Nc8-e7 Rg8-f8 15.Ne7-g8 Rc7-c8 16.Ng8-e7 Ne8-c7 17.Ne7-g8 Rf8-e8 18.Ng8-e7 Kg7-f8 19.Ne7-g8 Qh6-g7 20.Ng8-h6 Bd8-e7 21.Nh6-g8 Re8-d8 22.Ng8-h6 Kf8-e8 23.Nh6-g8 Be7-f8 24.Ng8-e7 Qg7-h6 25.Ne7-g8 Bf8-g7 26.Ng8-e7 Nh7-f8 27.Ne7-g8 Qh6-h7 28.Ng8-e7 Bg7-h6 29.Ne7-g8 Qh7-g7 30.Ng8-e7 Nf8-h7 31.Ne7-g8 Qg7-f8 32.Ng8-e7 Bh6-g7 33.Ne7-g8 Qf8-e7 34.Ng8-h6 Ke8-f8 35.Nh6-g8 Rd8-e8 36.Ng8-h6 Qe7-d8 37.Nh6-g8 Bg7-h6 38.Ng8-e7

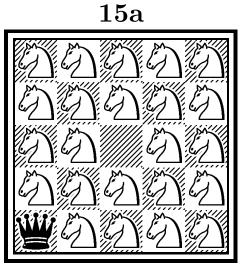
Kf8-g7 39.Ne7-g8 Re8-f8 40.Ng8-e7 Qd8-e8 41.Ne7-g8 Rc8-d8 42.Ng8-e7 Rf8-g8 43.Ne7-c8 Kg7-f8 44.Nc8-e7 Rg8-g7 45.Ne7-g8 Rd8-c8 46.Ng8-e7 Qe8-d8 47.Ne7-g8 Kf8-e8 48.Ng8-e7 Nh7-f8 49.Ne7-g8 Rg7-h7 50.Ng8-e7 Bh6-g7 51.Ne7-g8 Qd8-e7 52.Ng8-h6 Ke8-d8 53.Nh6-g8 Qe7-e8 54.Ng8-e7 Rh7-h6 55.Ne7-g8 Nf8-h7 56.Ng8-e7 Qe8-f8 57.Ne7-g8 Nc7-e8 58.Ng8-e7 Rc8-c7 59.Ne7-c8 Qf8-g8 60.Nc8-e7 Bg7-f8 61.Ne7-c8 Ne8-g7 62.Nc8-e7 Kd8-e8 63.Ne7-c8 Bf8-e7.

N13: Place white Queens on a1 and c1, a black Knight on a2, and a black Bishop on b2, as in N13a. The orthoreconstruction is unique modulo the direction of time: 1.Qa1-b1 Bb2-a1 2.Qc1-b2 Na2-c1 3.Qb2-a2 Ba1-b2 4.Qa2-a1 Nc1-a2 5.Qb1-c1.

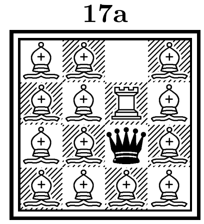
13a



15a



17a



N14: The shortest solution is unique modulo the direction of time: 1.Qb7-a7 Rb8-b7 2.Qa7-b8 Ra8-a7 3.Qb8-a8 Rb7-b8 4.Qa8-b7 Rb8-a8 5.Qb7-b8 Ra7-b7 6.Qb8-a7 Rb7-b8 7.Qa7-b7.

N15: Place the pieces as in N15a. The shortest orthoreconstruction is unique modulo the direction of time: 1.Na2-c3 Qa1-a2 2.Nb3-a1 Qa2-b3 3.Nb4-a2 Qb3-b4 4.Nc5-b3 Qb4-c5 5.Nd5-b4 Qc5-d5 6.Ne4-c5 Qd5-e4 7.Ne3-d5 Qe4-e3 8.Nd2-e4 Qe3-d2 9.Nc2-e3 Qd2-c2 10.Nb1-d2 Qc2-b1 11.Na1-c2 Qb1-a1 12.Nc3-b1.

N16: One of the 32 shortest solutions: 1. ... Qb2-c1 2.Nd1-b2 Qc1-d1 3.Bd2-

c1 Qd1-d2 4.Bc2-d1 Qd2-c2 5.Bc3-d2 Qc2-c3 6.Bb3-c2 Qc3-b3 7.Bb4-c3 Qb3-b4 8.Ba4-b3 Qb4-a4 9.Ba3-b4 Qa4-a3 10.Nb2-a4 Qa3-b2 11.Nb1-a3 Qb2-b1 12.Bc1-b2 Qb1-c1 13.Bc2-b1 Qc1-c2 14.Bb2-c1 Qc2-b2 15.Na3-c2 Qb2-a3 16.Na4-b2 Qa3-a4 17.Bb4-a3 Qa4-b4 18.Bb3-a4 Qb4-b3 19.Bc3-b4 Qb3-c3 20.Bc4-b3 Qc3-c4 21.Bd4-c3 Qc4-d4 22.Bd3-c4 Qd4-d3 23.Nc2-d4 Qd3-c2 24.Nb2-d3 Qc2-b2 25.Bb1-c2 Qb2-b1 26.Bc1-b2 Qb1-c1 27.Bc2-b1 Qc1-c2 28.Nd3-c1 Qc2-d3 29.Nd4-c2 Qd3-d4 30.Bc4-d3 Qd4-c4 31.Bc3-d4 Qc4-c3 32.Bb3-c4 Qc3-b3 33.Bb4-c3 Qb3-b4 34.Ba4-b3 Qb4-a4 35.Ba3-b4 Qa4-a3 36.Bb3-a4 Qa3-b3 37.Nc2-a3 Qb3-c2 38.Nc1-b3 Qc2-c1 39.Bb1-c2 Qc1-b1 40.Bb2-c1 Qb1-b2 41.Bc2-b1 Qb2-c2 42.Bc3-b2 Qc2-c3 43.Bd3-c2 Qc3-d3 44.Bd4-c3 Qd3-d4 45.Bc4-d3 Qd4-c4 46.Nb3-d4 Qc4-b3 47.Na3-c4 Qb3-a3 48.Ba2-b3 Qa3-a2 49.Bb2-a3 Qa2-b2 50.Bb3-a2 Qb2-b3 51.Nc4-b2 Qb3-c4 52.Nd4-b3 Qc4-d4 53.Bd3-c4 Qd4-d3 54.Bc3-d4 Qd3-c3 55.Bc2-d3 Qc3-c2 56.Bd2-c3 Qc2-d2 57.Bd1-c2 Qd2-d1 58.Bc1-d2 Qd1-c1 59.Nb2-d1 Qc1-b2 60.Nb3-c1 Qb2-b3 61.Ba3-b2 Qb3-a3 62.Ba2-b3 Qa3-a2 63.Bb2-a3 Qa2-b2 64.Nc1-a2 Qb2-c1 65.Nd1-b2 Qc1-d1 66.Bd2-c1 Qd1-d2 67.Bc2-d1 Qd2-c2 68.Bc3-d2 Qc2-c3 69.Bd3-c2 Qc3-d3 70.Bd4-c3 Qd3-d4 71.Bc4-d3 Qd4-c4 72.Bc3-d4 Qc4-c3 73.Nb2-c4 Qc3-b2 74.Na2-c3 Qb2-a2 75.Ba1-b2 Qa2-a1 76.Bb1-a2 Qa1-b1 77.Bb2-a1 Qb1-b2 78.Nc3-b1 Qb2-c3 79.Nc4-b2 Qc3-c4 80.Bd4-c3 Qc4-d4 81.Bd3-c4 Qd4-d3 82.Bc3-d4 Qd3-c3 83.Bc2-d3 Qc3-c2 84.Bd2-c3 Qc2-d2 85.Bd1-c2 Qd2-d1 86.Bc1-d2 Qd1-c1 87.Nb2-d1 Qc1-b2.

N17: Place the pieces as in N17a. One of the four shortest orthoreconstructions: 1.Rc3-c4 Qc2-c3 2.Bb3-c2 Qc3-b3 3.Rc4-c3 Qb3-c4 4.Rc3-b3 Qc4-c3 5.Bd3-c4 Qc3-d3 6.Bd2-c3 Qd3-d2 7.Bc2-d3 Qd2-c2 8.Bc1-d2 Qc2-c1 9.Bb1-c2 Qc1-b1 10.Bb2-c1 Qb1-b2 11.Bc2-b1 Qb2-c2 12.Rb3-b2 Qc2-b3 13.Rb2-c2 Qb3-b2 14.Ba2-b3 Qb2-a2 15.Ba3-b2 Qa2-a3 16.Bb3-a2 Qa3-b3 17.Bb4-a3 Qb3-b4 18.Bc4-b3 Qb4-c4 19.Bc3-b4 Qc4-c3 20.Bb3-c4 Qc3-b3 21.Rc2-c3 Qb3-c2 22.Bc4-b3.

N18: At least one of the Bishops needs to be able to traverse an odd cycle, so the first move must be to either d4 or e5. If 1.Be3-d4, then one of the sides will be forced to make a capture by the sixth single move. Therefore, 1. ... Nd3-e5 and then 2.Be4-d3 Nc5-e4 3.Be3-c5 Nd5-e3 4.Bc4-d5 Nd6-c4 5.Bc5-d6 Ne4-c5 6.Bd3-e4 Ne5-d3 7.Bd6-e5 Nc4-d6 8.Bd5-c4 Nf4-d5 9.Be5-f4 Nd3-e5 10.Bc4-d3 Ne3-c4 11.Bf4-e3 Nd5-f4 12.Be4-d5 Nc5-e4 13.Be3-c5 Nc4-e3 14.Bd5-c4 Ne3-d5 15.Bc5-e3 Ne4-c5 16.Bd3-e4 Ne5-d3.

N19: The two bRs cannot switch places, so the two bNs have to. Modulo the direction of time, the switch requires one bN to wait on g8 while the other one is transferred from h7 to e8 via f6. While the second bN occupies f6, the empty space needs to be transfered from h7 to f8. Therefore, the bQ has to be extracted from f5 before that (so that she can play the move f8-g7), the bK taking her place.

Once worked out, this plan is implemented without much difficulty. One of the 192 shortest solutions, White only playing his Qc8: 1.Rd8-e8 2.Kh7-g7 3.Nf6-h7 4.Ng8-f6 5.Rf8-g8 6.Re8-f8 7.Nf6-e8 8.Nh7-f6 9.Kg7-h7 10.Ne8-g7 11.Rf8-e8 12.Rg8-f8 13.Nf6-g8 14.Qf5-f6 15.Ng7-f5 16.Qf6-g7 17.Ng8-f6 18.Qg7-g8 19.Kh7-g7 20.Nf6-h7 21.Kg7-f6 22.Nf5-g7 23.Kf6-f5 24.Nh7-f6 25.Qg8-h7 26.Rf8-g8 27.Re8-f8 28.Ng7-e8 29.Rg8-g7 30.Qh7-g8 31.Rg7-h7 32.Qg8-g7 33.Nf6-

g8 34.Ne8-f6 35.Rf8-e8 36.Qg7-f8 37.Rh7-g7 38.Nf6-h7 39.Ng8-f6 40.Rg7-g8 41.Qf8-g7 42.Re8-f8 43.Nf6-e8 44.Nh7-f6 45.Qg7-h7 46.Ne8-g7 47.Rf8-e8 48.Rg8-f8 49.Nf6-g8 50.Kf5-f6 51.Ng7-f5 52.Kf6-g7 53.Ng8-f6 54.Qh7-g8 55.Kg7-h7 56.Qg8-g7 57.Rf8-g8 58.Re8-f8 59.Nf6-e8 60.Qg7-f6 61.Nf5-g7 62.Qf6-f5 63.Ne8-f6 64.Ng7-e8 65.Kh7-g7 66.Nf6-h7 67.Ne8-f6 68.Rf8-e8 69.Rg8-f8 70.Nf6-g8 71.Nh7-f6 72.Kg7-h7 73.Re8-d8.

N20: The shortest solution is unique modulo the direction of time: 1.Qa2-b3 Bb1-a2 2.Rb2-b1 Ba1-b2 3.Rb1-a1 Ba2-b1 4.Ra3-a2 Bb2-a3 5.Ra2-b2 Bb1-a2 6.Rb2-b1 Ba3-b2 7.Qb3-a3 Ba2-b3 8.Ra1-a2 Bb2-a1 9.Ra2-b2 Bb3-a2 10.Rb2-b3 Ba1-b2 11.Rb1-a1 Ba2-b1 12.Qa3-a2 Bb2-a3 13.Qa2-b2 Bb1-a2 14.Qb2-b1 Ba3-b2 15.Rb3-a3 Ba2-b3 16.Ra1-a2 Bb2-a1 17.Ra2-b2 Bb3-a2 18.Rb2-b3 Ba1-b2 19.Qb1-a1 Ba2-b1 20.Qa1-a2 Bb2-a1 21.Rb3-b2.

N21: Black can easily reconstruct the north-west corner in an odd number of moves by playing Ba7-b8... Qa6-a7... Qb6-a6... Qa7-b6... Bb8-a7. Alas, the white Queen must intervene before and after every one of these moves, triggering a long follow-the-leader sequence across the north and effectively slowing the process down by a factor of seven.

The shortest solution is unique modulo the direction of time: 1.Qb7-b8 Nd6-b7 2.Ne8-d6 Rf8-e8 3.Bg7-f8 Qh8-g7 4.Rh7-h8 Qg7-h7 5.Bf8-g7 Re8-f8 6.Nd6-e8 Nb7-d6 7.Qb8-b7 Ba7-b8... 14. ... Qa6-a7... 21. ... Qb6-a6... 28. ... Qa7-b6... 35. ... Bb8-a7... 42.Qb8-b7.

N22: The only way to extract a piece from the box is to lead the wQ to f7 with a bN on e6 and then play, with Black on the move, Ne6-f8 Qf7-e6+ Nf8-d7... Therefore, the game must open with 1. ... Nd8-e6 2.Kg8-h7 Ne6-d8 3.Rf8-g8 Nd8-e6 4.Bg7-f8 Ne6-d8 5.Rg8-g7 Nd8-e6 6.Kh7-g8 Ne6-d8 7.Rg7-h7 Nd8-e6 8.Bf8-g7 Ne6-f8 9.Qf7-e6+ Nf8-d7...

A change of parity is now easily attained. Once we have pulled the box apart, though, we need to put it back together in order to complete the solution – and, in this problem, this cannot be done by simply playing out the extraction in reverse.

The easiest way to see how to put all white pieces back into the box is to go looking for a way to extract a piece when playing *backwards* from the diagram. As it turns out, in reverse time the extracted piece needs to be the King, with a similar accompanying manoeuvre by the black Knight.

There are many ways to cobble the two ends of the solution together, such as 10.Qe6-f7 Nd7-b8 11.Qf7-f8 Nb8-d7 12.Kg8-f7 Nd7-b8 13.Kf7-e6 Nb8-d7 14.Qf8-f7 Nd7-b8 15.Qf7-g8 Nb8-d7 16.Bg7-f8 Nd7-b8 17.Rh7-g7 Nb8-d7 18.Qg8-h7 Nd7-b8 19.Rg7-g8 Nb8-d7 20.Bf8-g7 Nd7-f8+ 21.Ke6-f7 Nf8-e6...

All white pieces gathered back inside the box (and the black Knight lock put back in place), we are only left to re-create the diagram position: 22.Rg8-f8 Ne6-d8+ 23.Kf7-g8 Nd8-e6 24.Rf8-f7 Ne6-d8 25.Bg7-f8 Nd8-e6 26.Qh7-g7 Ne6-d8 27.Kg8-h7 Nd8-e6 28.Qg7-g8 Ne6-d8 29.Bf8-g7 Nd8-e6 30.Rf7-f8 Ne6-d8 31.Qg8-f7 Nd8-e6 32.Kh7-g8 Ne6-d8. This is one of the shortest solutions; due to the loose second act, there are many others.